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A study of turbulent heat transfer in a sudden-expansion pipe with drag-reducing viscoelastic fluid

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Abstract—The turbulent heat transfer behavior of a drag-reducing viscoelastic fluid is experimentally investigated in a sudden-expansion circular pipe with aqueous solutions of polyacrylamide (200, 500, and 1000 ppm). The ranges of Reynolds and Prandtl numbers tested are 6200–63 000 and 8.5–16.2, respectively. A minimum asymptote of the local maximum Nusselt number is found to exist at the sudden-expansion pipe flow with the drag-reducing viscoelastic fluid and is given by the following equation: $Nu_{x,\max} = 0.233Re_{a,d}^{0.83}$. When the average Nusselt number is compared with and without the sudden-expansion step, the percentage heat transfer enhancement due to the sudden-expansion step for the polyacrylamide solutions is 40–63%, depending on the Reynolds number, while that for water is 3–6%. The current results suggest that many energy-intensive industries that process huge amounts of viscoelastic fluids through various types of heat exchangers can capitalize upon heat transfer performance enhanced by the sudden-expansion step.

INTRODUCTION

WHEN FLOW separation occurs in pipes or ducts, the flow in and after the separation region becomes very complex and the heat transfer characteristics are often significantly altered by the nature of the flow separation and subsequent flow redevelopment. Such flow separations are found in various engineering problems such as sudden expansions and contractions, in rapidly diverging sections, and upstream and downstream of orifices. Turbulent heat transfer behavior of a Newtonian flow across a sudden-expansion step has been of technical interest due to its potential as a tool to enhance convective heat transfer performance in various types of heat exchangers. Most of the previous works reported in the literature on this subject have dealt with Newtonian fluids such as air and water.

The study of real fluids used in the chemical, pharmaceutical, food, and biomedical industries has become increasingly important recently, largely due to severe limitations in the application of Newtonian flow theories to industrial flow problems. A better understanding of non-Newtonian flows through sudden-expansion pipes and ducts should lead to the design and development of more energy-efficient processes and to better quality control of the final products.

As shown in the results of the previous investigations of abrupt expansion pipe flows with Newtonian fluids, the local heat transfer coefficient near the reattachment region increases significantly from the fully developed flow value. However, it decreases rapidly to the fully developed value due to the relatively short hydrodynamic and thermal entrance length (i.e. 20–30 pipe diameters) of a Newtonian fluid flow, which is typical of turbulent flow. In contrast, for drag-reducing viscoelastic fluids flowing under turbulent flow conditions, the thermal entry length was reported to be extremely long—approximately 600–1000 pipe diameters—while the corresponding hydrodynamic entry length was only 80–100 [1–3]. Hence, the long thermal entrance length of drag-reducing viscoelastic fluids might be used as a tool for convective heat transfer enhancement by a sudden step in pipe and duct flows, while the elasticity of such fluids minimizes the increase in the turbulent friction coefficient.

To verify this concept in practical heat transfer problems, it is necessary to understand the fluid dynamics and heat transfer characteristics of these fluids in flow geometries more complex than a straight circular pipe such as a sudden-expansion pipe section. The purposes of the present study were to better understand the convective heat transfer characteristics of turbulent flows in an abrupt expansion tube containing separated and reattached regions and to examine whether the concept of using a downward-facing step together with a drag-reducing viscoelastic fluid lent valid heat transfer enhancement. The current study reports heat transfer data, while the hydrodynamic characteristics of the drag-reducing viscoelastic fluid across the sudden-expansion step will be reported elsewhere [4].

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NOMENCLATURE

- C_p specific heat
- d pipe diameter of hydrodynamic entry section
- D pipe diameter of heat transfer test section
- f Fanning friction coefficient
- h local heat transfer coefficient
- H step height, $(D-d)/2$
- k thermal conductivity of test fluids
- $\dot{m}$ mass flow rate
- Nu Nusselt number, hD/k
- $Nu_{x,\max}$ maximum Nusselt number at reattachment point
- q heat flux
- Q total heat generated in entire test pipe
- Q_x heat generated in test pipe of length x
- Re_a Reynolds number based on apparent viscosity, η_a
- T temperature
- U average velocity at heat transfer test section.

Greek symbols
- $\dot{\gamma}$ shear rate
- η_a apparent viscosity as a function of shear rate
- κ_s thermal conductivity of tube
- λ characteristic time of viscoelastic fluid
- μ viscosity of Newtonian fluids, independent of shear rate
- τ shear stress.

Subscripts
- a property based on apparent viscosity, η_a
- b bulk
- fd fully developed conditions in straight pipe flow
- i inner wall
- in inlet condition
- o outer wall
- out outlet condition
- x local
- w wall.

BACKGROUND

Non-Newtonian fluid flow in straight circular pipe

Aqueous solutions of drag-reducing viscoelastic fluids flowing through a circular pipe under turbulent flow conditions produce significant friction reduction when compared with the values produced with a pure solvent (i.e. water). The extent of friction and heat transfer reductions is ultimately limited by unique minimum asymptotes, which are independent of the polymer concentration, the solvent chemistry, and the degree of degradation. The minimum asymptotes are solely dependent on the Reynolds number and the dimensionless axial distance x/d. For both hydrodynamically and thermally fully developed turbulent flows, the following friction and heat transfer minimum asymptotes have been observed [2]:

$$f = 0.20 Re_a^{-0.48} \quad (1)$$

$$j_H = 0.03 Re_a^{-0.45}. \quad (2)$$

Note that f and j_H for drag-reducing viscoelastic fluids also depend on the Weissenberg number, defined as $\lambda U/D$. However, there exist critical values of the Weissenberg number for friction and heat transfer [2] above which f and j_H are functions only of the Reynolds number, as shown above. Also note that for drag-reducing viscoelastic fluids, the percentage turbulent heat transfer reduction would always be greater than the percentage friction reduction, if one had thermally and hydrodynamically fully developed flows. However, in a finite-length tube (as is seen in many heat exchanger designs), the entire heat transfer test section may be under the influence of thermal entry flow due to the extremely long thermal entrance length of drag-reducing viscoelastic fluids.

Separated flow of a Newtonian fluid

Due to the recent development of high-performance thermal systems that use a forced flow separation to improve convective heat transfer, there has been an increasing need for more quantitative information on friction and heat transfer coefficients in separated flow regions. Krall and Sparrow [5] reported experimental results for turbulent heat transfer over the flow in the separated region induced by an orifice in a circular pipe. They used water as their working fluid and experimented with four different size orifices (i.e. 0.25, 0.33, 0.50, and 0.67 tube diameters) over a range of Reynolds numbers, from 10 000 to 130 000, and at Prandtl numbers of 3 and 6. In particular, they reported the following correlation, which implied the Reynolds number dependency of the local maximum Nusselt number, $Nu_{x,\max}$:

$$Nu_{x,\max} = 0.398 Re_d^{2/3} \quad (3)$$

where Re_d is the Reynolds number based on the upstream pipe diameter, d. They also found that at the reattachment point occurring at 1.25–2.25 diameters from the onset of separation, the local heat transfer coefficients were three to nine times greater than those corresponding to the fully developed values, while they remained almost independent from the Prandtl number.

Zemanick and Dougall [6] presented experimental results of turbulent heat transfer in abrupt expansion pipe flow. With air as the working fluid, their experiments were performed at three different expansion geometries, i.e. $D/d = 1.22$, 1.85, and 2.33. In particular, they reported a fairly consistent although weak Reynolds-number dependency concerning the relative maximum Nusselt number, $Nu_{x,\max}/Nu_{fd}$, for air flow. In contrast, Krall and Sparrow [5] clearly showed important Reynolds-number dependence in the relative Nusselt number for water. Zemanick and Dougall [6] presented the local maximum Nusselt number proportional to the Reynolds number of the 2/3-power, similar to Krall and Sparrow's observation, i.e. equation (3), for water

$$Nu_{x,\max} = 0.20Re_d^{2/3}. \quad (4)$$

Separated flow of a non-Newtonian fluid

The fundamental hydrodynamics of sudden contraction and gradually converging nozzle flows with non-Newtonian fluids have been investigated by many researchers [7–14]. In contrast, corresponding studies of sudden-expansion flow are rarely found [15, 16] and are conducted in the laminar flow region (i.e. $Re < 150$) due to an intended application to the polymer processing field. Halmos *et al.* [15] found that for a power-law fluid (i.e. inelastic fluid) the reattachment length increased with an increasing Reynolds number at a given power-law index, n. Also, as the power-law index n decreased from 1.0 to 0.65, the reattachment length and the size of the secondary cell increased by 20% in the range of Reynolds number ($Re < 150$). The effect of the expansion ratio on the reattachment length was numerically studied at $Re = 10$, which showed the trend toward increasing reattachment lengths for increasing expansion ratios [15].

Perera and Walters [16] demonstrated numerically that elasticity definitely reduced the size of the vortex in a sudden-expansion pipe in the laminar flow regime. They also found that inelastic fluid had a large recirculating vortex in the sudden-expansion flow, while the vortex almost disappeared in elastic fluid under identical flow conditions, demonstrating the influence of elasticity in the sudden-expansion flow. Recently, Pak *et al.* [17] conducted a flow visualization study to investigate the hydrodynamic characteristics of a drag-reducing viscoelastic fluid (e.g. aqueous solutions of polyacrylamide) across a sudden-expansion step over a wide range of Reynolds numbers (e.g. 10–35 000 based on the upstream diameter) including laminar, transition, and turbulent flows. The reattachment lengths for polyacrylamide solutions in the laminar flow regime were found to be much shorter than those for the Newtonian fluid. In addition, they decreased significantly with increasing concentrations of the polyacrylamide solutions at the same Reynolds number. Furthermore, in the turbulent flow regime, the reattachment length for the polyacrylamide solutions was two or three times longer than those for water and gradually increased with increasing concentrations of polyacrylamide solutions, resulting in 25 and 28 step-height distances for 500 and 1000 ppm concentrations, respectively.

EXPERIMENTAL METHOD AND FACILITY

A schematic diagram of the stainless steel flow loop is shown in Fig. 1. The loop includes a 500 gal stainless steel tank, a pump, a bypass line, a surge tank, a calming chamber, a hydrodynamic entry section, a main heat transfer test section, a mixing chamber, and a fluid collection apparatus for measuring and calibrating flow rates.

Two different expansion ratios (D/d) were used by changing the diameter of the hydrodynamic entry section, which resulted in values of 1.391 and 1.899. In both cases, the inside diameter of the sudden-expansion test section was $D = 1.262$ cm. The lengths of the hydrodynamic entry tubes used in the current study were 121.9 cm ($L/d = 134.4$) for the small expansion ratio case ($D/d = 1.391$) and 91.4 cm ($L/d = 137.6$) for the large expansion ratio case ($D/d = 1.899$), respectively. Both should yield a fully developed velocity profile at the entrance of the sudden-expansion test section. To provide a downward-facing sudden-expansion step, the hydrodynamic entry tube was inserted 1.27 cm into the heat transfer test tube to ensure the symmetry and alignment of the sudden-expansion section. A total of 41 thermocouples were mounted on the outside surface of the test section for temperature measurements. At four different axial locations, four thermocouples were installed circumferentially to check circumferential temperature variations during tests. Detailed locations of these thermocouples are given elsewhere [18].

To obtain a constant heat flux boundary condition, the heat transfer test section, with its length of $x/D = 225.1$, was heated electrically by a constant d.c. power supplier capable of delivering a maximum of 24 kW at a maximum of 12 V. To minimize parasitic heat loss due to axial heat conduction, the test section was isolated electrically and thermally from its upstream and downstream sections by Nylon bushings. To avoid convective heat loss to the surroundings, the heat transfer section, as well as the calming and mixing chambers, were wrapped with 7.6-cm-thick fiberglass insulation blankets. The remaining parts of the flow loop, except for the hydrodynamic entry and main heat transfer test sections, were fabricated from stainless steel tube with an outer diameter of 2.54 cm.

EXPERIMENTAL PROCEDURES AND DATA REDUCTION

Aqueous solutions of polyacrylamide (Separan AP-273 : 200, 500, and 1000 ppm by weight) were used to investigate the heat transfer characteristics of drag-reducing viscoelastic fluids in the sudden-expansion tube. Before the heat transfer tests with the polyacrylamide solutions, system calibration runs were

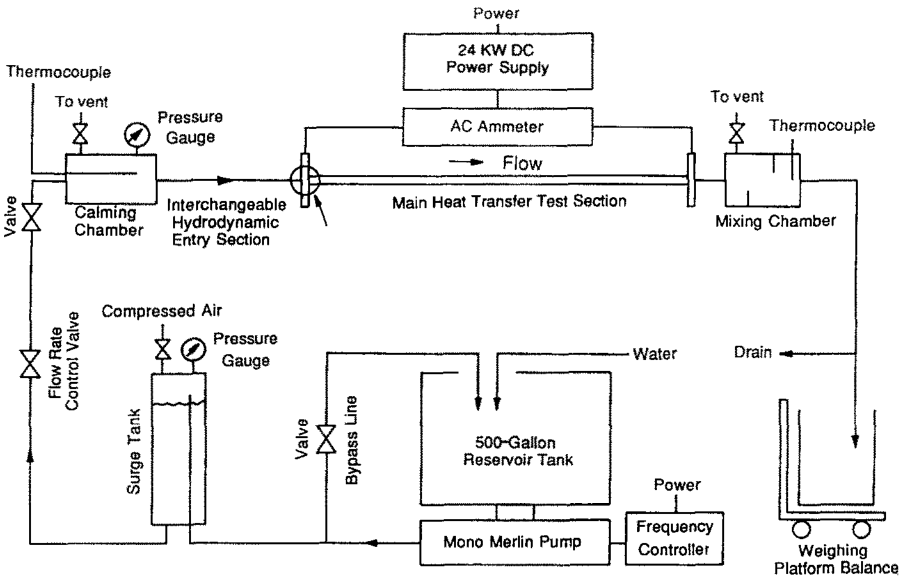

FIG. 1. Schematic diagram of heat transfer flow loop.

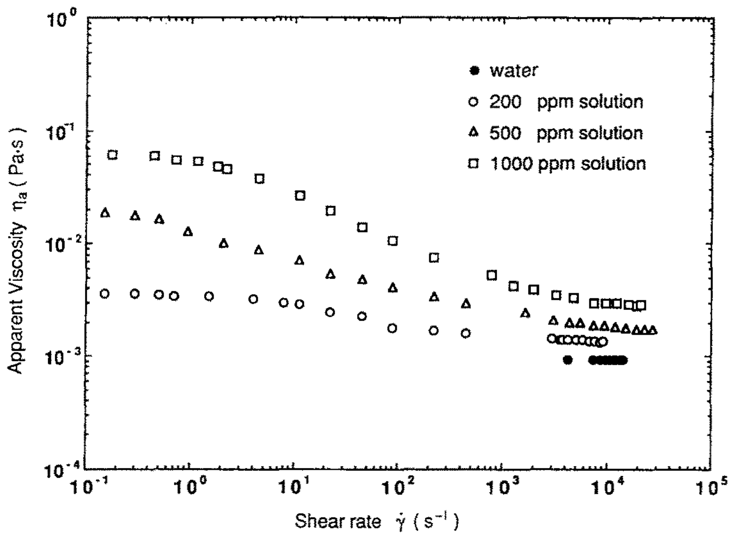

FIG. 2. Apparent viscosity vs average shear rate for water and polyacrylamide (Separan AP-273) solutions.

performed with tap-water to check the validity of the experimental apparatus and overall experimental procedure.

Viscosities of aqueous solutions were measured with a capillary tube viscometer, a Brookfield viscometer, and a falling-needle viscometer to cover a wide range of shear rates. Test fluid samples collected from the outlet of the flow loop during experiments were used for viscosity measurements. Figure 2 shows the viscosities of the working fluids used for the experiments. The density, the specific heat, and the thermal conductivity of aqueous solutions of polyacrylamide were reported to be almost the same as those obtained for water—within 3% of the values of water at a given temperature of interest [19]. In view of the small difference, which may have been due to experimental error, those values for water were used for the present polymer solutions.

The heat generated in the test section was determined by measuring both the voltage drop across the heat transfer test section and the electric current, together with the electrical resistance of the stainless steel tube. The value was double-checked calorimetrically by measuring the inlet and outlet fluid bulk temperatures of the test fluids. The maximum error margin was approximately 2.5% for the calibration test with water. Uniform heat flux conditions were ensured by measuring and comparing circumferential temperatures at four locations along the test section.

Note that the small temperature difference between inlet and outlet, as well as between wall and bulk temperatures, was important in minimizing the effect of temperature-dependent viscosity on the calculation of the overall heat transfer coefficient. The maximum temperature difference between the inlet temperature of test fluids and the inner wall temperature at the exit (i.e. at $x/D = 225.1$) was less than 6°C for all runs. The apparent viscosity of a dilute aqueous non-Newtonian solution generally decreases 5–7% depending on its concentration as the temperature of the solution increases from 20 to 25°C [20]. For example the value of $d\eta/dT$ is -0.2 Pa s °C^{-1} for distilled water and -0.13 Pa s °C^{-1} for CMC 1000 ppm solution in the temperature range of 20–30°C.

The essential quantity to be measured during a test was outer wall temperatures along the heat transfer test section at 30 different locations. Also measured were wall and bulk temperatures, mass flow rates, electric power inputs, and pressure drops along the test section. In particular, the inlet and outlet bulk temperatures were measured in a calming chamber and in a mixing chamber, respectively. To ensure the correct measurement of the outlet bulk temperature, two baffles were used inside the mixing chamber (as shown in Fig. 1). Physical properties (e.g. thermal conductivity, density, and specific heat) of the test tube needed for data reduction were taken from the earlier studies [2, 19]. The local heat transfer coefficient, h_x, is obtained from local quantities

$$h_x = q/(T_{i,w} - T_b) \tag{5}$$

in which q is the rate of heat flux at the wall, $T_{i,w}$ the inside wall temperature, and T_b the bulk temperature. Because $T_{i,w}$ could not be measured directly for an electrically heated tube in practice, equation (5) may be rearranged in a different form

$$h_x = \frac{q}{(T_{o,w} - T_{in}) - (T_{o,w} - T_{i,w}) - (T_b - T_{in})}. \tag{6}$$

The temperature difference $(T_{o,w} - T_{in})$ can be measured directly, and the second and third terms in the denominator in equation (6) could be calculated from the heat conduction equation in the cylindrical coordinates and from the energy balance, respectively, as

$$T_{o,w} - T_{i,w} = \frac{Q[2D_o^2 \ln(D_o/D_i) - (D_o^2 - D_i^2)]}{4\pi(D_o^2 - D_i^2)\kappa_s L} \tag{7}$$

and

$$T_b - T_{in} = \frac{Q_x}{\dot{m}C_p} \tag{8}$$

where κ_s is the thermal conductivity of the tube. The temperature difference given in equation (7) was obtained under the assumption of an insulated boundary condition at the outer wall and a convective boundary condition at the inner wall of the tube. Once the local heat transfer coefficient h_x was determined, a local Nusselt number was calculated from

$$Nu_x = h_x D/k \tag{9}$$

where k is the thermal conductivity of test fluids (which was assumed to be constant for small inlet and outlet bulk temperature differences). In addition, the Reynolds and Prandtl numbers were evaluated from their respective definitions

$$Re_a = 4\dot{m}/\eta_a \pi D \tag{10}$$

and

$$Pr_a = C_p \eta_a/k \tag{11}$$

in which $\dot{m}$ is the mass flow rate, and the apparent viscosity η_a is defined as $\tau_w/\dot{\gamma}_w$.

The local Nusselt number, Nu_x, was plotted against the dimensionless axial distance, x/D, and was also compared with fully developed turbulent heat transfer results in the circular tube as follows:

$$Nu_x = f(x/D, Re_a) \tag{12}$$

and

$$Nu_x/Nu_{fd} = f(x/D, Re_a) \tag{13}$$

where Nu_{fd} is the value corresponding to the fully developed region of the heat transfer test section.

RESULTS AND DISCUSSION

To validate our experimental system and data reduction procedure, calibration runs were conducted with water. The Nusselt numbers obtained at the fully redeveloped region of the heat transfer test section are shown in Fig. 3. The results shown by open circles were obtained from the sudden-expansion ratio $D/d = 1.391$, while the results shown by closed circles

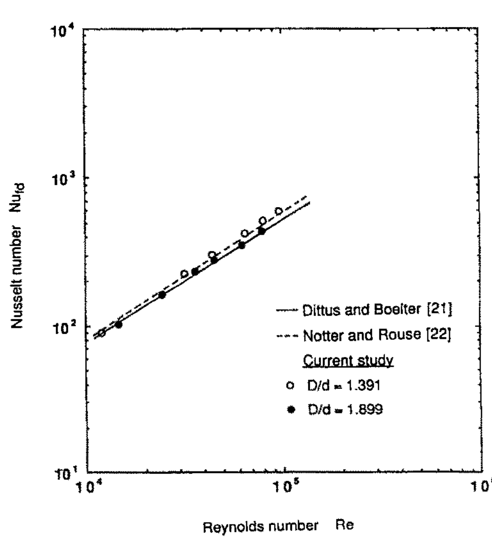

FIG. 3. Nusselt numbers at fully developed region of a sudden-expansion section for water.

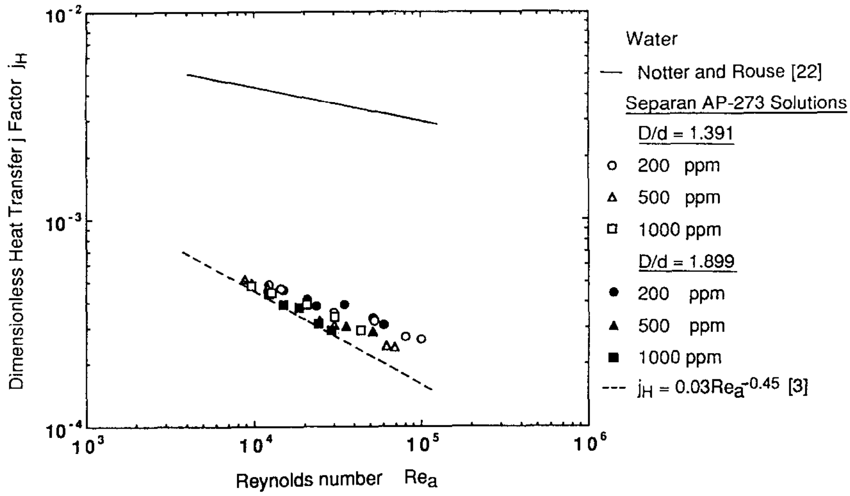

FIG. 4. Dimensionless heat transfer j factor, j_H, at $x/D = 225.1$ for polyacrylamide solutions.

were obtained from the sudden-expansion ratio $D/d = 1.899$. The Nusselt number results obtained in the fully developed region of the heat transfer test section for water agree well with the previous correlations [21, 22].

The heat transfer results corresponding to the fully developed flows for the 200, 500, and 1000 ppm polyacrylamide solutions are given in Fig. 4 in the form of the dimensionless heat transfer j factor j_H. Again, the open and closed circles are from the two different expansion ratios of 1.391 and 1.899, respectively. The heat transfer j factors for these polyacrylamide solutions show significant heat transfer reduction even at 200 ppm, approaching the 'minimum heat transfer asymptote' (equation (2)), shown as a dashed line. Note that results for the 1000 ppm solution agree well with predictions from the minimum heat transfer asymptote.

Representative inner wall and bulk temperature distributions are shown in Figs. 5(a)–(c). Figure 5(a) shows the local inner wall temperature profile for tapwater. Also shown is a straight line connecting the two bulk temperatures measured in the inlet and outlet mixing chambers. For water, a thermally fully developed region is clearly obtained beyond approximately 40 tube diameters from the sudden-expansion step as it is manifested by two parallel lines. In general, within the separated region (i.e. $x/D < 2$), the wall-to-bulk temperature difference decreases with increasing x/D until a minimum value is reached (presumably at the reattachment point, which is not shown clearly due to a long heat transfer test section). Then, with further increases in x/D, the temperature difference increases until a thermally developed state is reached (i.e. at $x/D = 15$). Figures 5(b) and (c) present the local inner wall and bulk temperatures for the 200 and 1000 ppm solutions of polyacrylamide, respectively. For these solutions, the thermally fully developed flows are not established even at the end of the heat transfer test section (i.e. $x/D = 225.1$).

Newtonian fluid

The turbulent heat transfer test results for water with two different expansion ratios ($D/d = 1.391$ and 1.899) are presented in the form of the Nusselt number vs the dimensionless axial distance at various Reynolds numbers in Figs. 6 and 7. The relative Nusselt number, Nu_x/Nu_{fd}, is given in the insets of these figures so that the local heat transfer enhancement (relative to the fully developed value) due to the sudden-expansion step can be examined.

In the case of the expansion ratio of 1.391 shown in Fig. 6, the local Nusselt number initially increases in the flow separation region, reaching a maximum value at $x/D < 2$ and then gradually decreases to an asymptotic value at each Reynolds number. The local maximum Nusselt numbers at all six different Reynolds number cases occur at $x/D = 1.01$–1.26, which is equal to $x/H = 7.17$–9.00 in terms of step height (H). The local Nusselt number in general increases with an increasing Reynolds number, which is typical of turbulent heat transfer characteristics. At $x/D \cong 20$, the local Nusselt numbers remain constant for all Reynolds number cases, which suggests that the thermally fully developed flow is established after that point for water at $D/d = 1.391$.

The relative maximum Nusselt number, $Nu_{x,max}/Nu_{fd}$, for $D/d = 1.391$ as is shown in the inset of Fig. 6, is approximately 1.6–2.5. With the increasing Reynolds number, the maximum relative Nusselt number decreases, which is an opposite trend in comparison to the local Nusselt number trend. This suggests that the relative increase in heat transfer coefficients due to a sudden-expansion step is greater for water at a smaller Reynolds number.

Similar results for water with another expansion ratio, $D/d = 1.899$, are shown in Fig. 7. The locations of maximum heat transfer occur at $x/D = 1.33$–1.77, which is equal to $x/H = 9.51$–12.68. It should be noted that the maximum relative Nusselt number, $Nu_{x,max}/Nu_{fd}$, for $D/d = 1.899$, is 2.3–3.4, which is larger

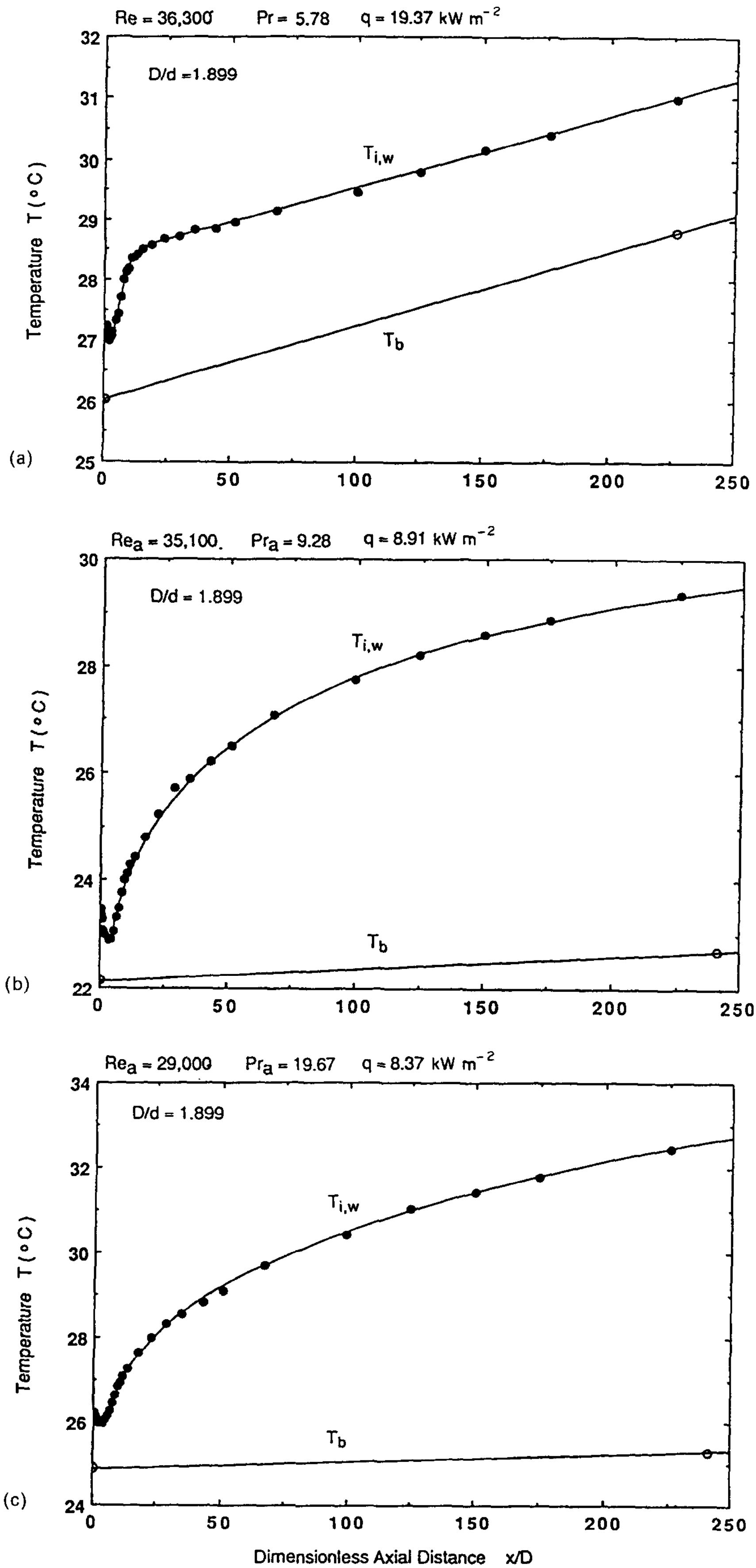

FIG. 5. Representative inner wall and bulk temperature profiles along the heat transfer test section: (a) water, (b) 200 ppm polyacrylamide solution, and (c) 1000 ppm polyacrylamide solution.

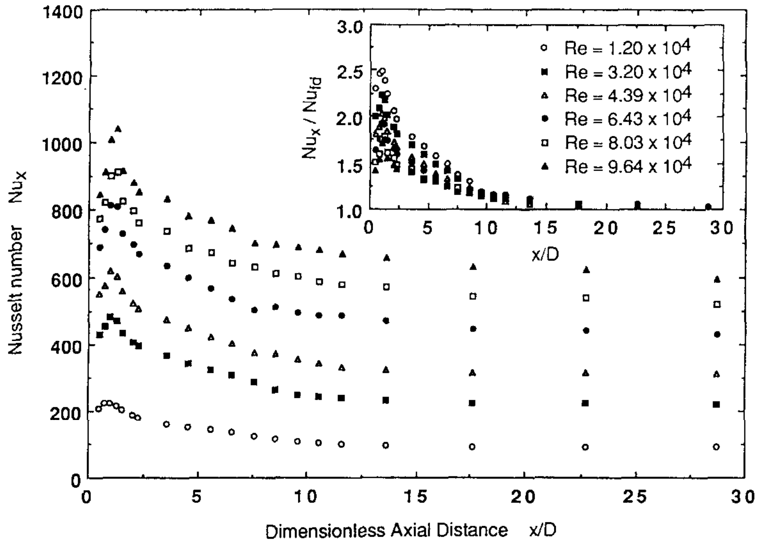

FIG. 6. Turbulent heat transfer results for water ($D/d = 1.391$). Inset shows relative local Nusselt number vs x/D.

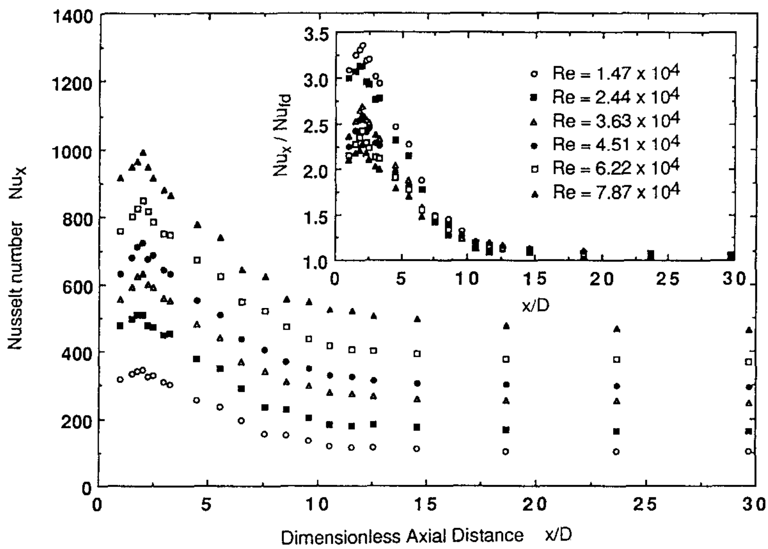

FIG. 7. Turbulent heat transfer results for water ($D/d = 1.899$). Inset shows relative local Nusselt number vs x/D.

than that for the smaller expansion ratio case (i.e. 1.6–2.5 for $D/d = 1.391$).

Drag-reducing viscoelastic fluids

Turbulent heat transfer test results for drag-reducing viscoelastic fluids were obtained using aqueous solutions of polyacrylamide (200, 500, and 1000 ppm). Here, the results for 200 and 1000 ppm solutions with two different expansion ratios ($D/d = 1.391$ and 1.899) are presented in Figs. 8–11. The relative Nusselt numbers are also shown in the inset of each figure for comparison to the fully developed heat transfer data. Because the thermally developing state continues to the last data point (i.e. at $x/D = 225.1$, as shown in Figs. 5(b)–(c)) with polyacrylamide solutions, the local Nusselt number at that point (i.e. $Nu_{x/D=225.1}$) is used in the calculation of the relative Nusselt number.

For the 200 ppm polyacrylamide solution, the heat transfer test results are presented in Figs. 8 and 9. The former is for $D/d = 1.391$, and the latter is for $D/d = 1.899$. As shown in these figures, the local maximum heat transfer occurs at x/D equal to about 2.27–4.53. It should be mentioned that the location of the local maximum Nusselt number moves downstream of the sudden-expansion section when compared to the water data. This implies that the reat-

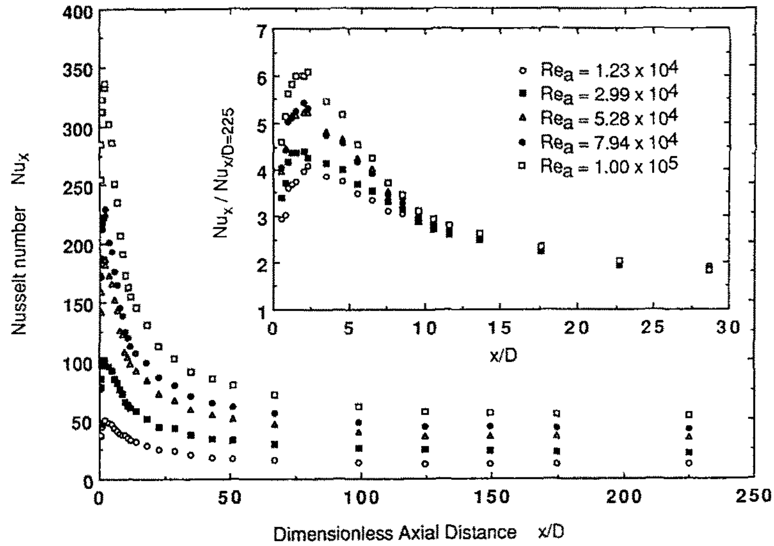

FIG. 8. Turbulent heat transfer results for 200 ppm polyacrylamide solution ($D/d = 1.391$). Inset shows relative local Nusselt number vs x/D.

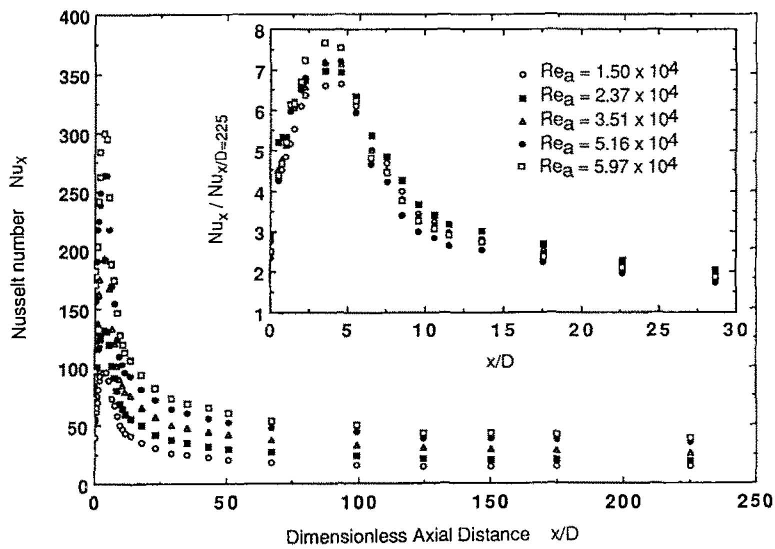

FIG. 9. Turbulent heat transfer results for 200 ppm polyacrylamide solution ($D/d = 1.899$). Inset shows relative local Nusselt number vs x/D.

tachment length in an abrupt expansion pipe flow with polyacrylamide solutions is longer than that with Newtonian fluid for the turbulent flow. Note that for the laminar flow the reattachment length is much shorter for viscoelastic fluid flows than for water [16, 17]. The local Nusselt number increases with an increasing Reynolds number, which is similar to the trends observed in water cases. However, the values of the relative maximum Nusselt number (shown in the inset of Fig. 8) increase with an increasing Reynolds number, which shows a trend opposite to that for water.

The effect of expansion ratio (D/d) on relative heat transfer enhancement can clearly be demonstrated by comparing the results in Figs. 8 and 9 for the 200 ppm polyacrylamide solution. The magnitude of the relative maximum Nusselt number for the $D/d = 1.899$ case is 6.5–7.8, depending on Reynolds numbers, while that for the $D/d = 1.391$ case is 4.1–6.2. More specifically, at a Reynolds number of 5.2×10^4, the relative maximum Nusselt number increases from 5 to 7 as the expansion ratio increases from 1.391 to 1.899 (see insets of Figs. 8 and 9). Note that the relative Nusselt number for the 200 ppm polyacrylamide solution is two to three times larger than that for water at the same Reynolds number and expansion ratio. This suggests that the use of a sudden-expansion step is of considerably more benefit

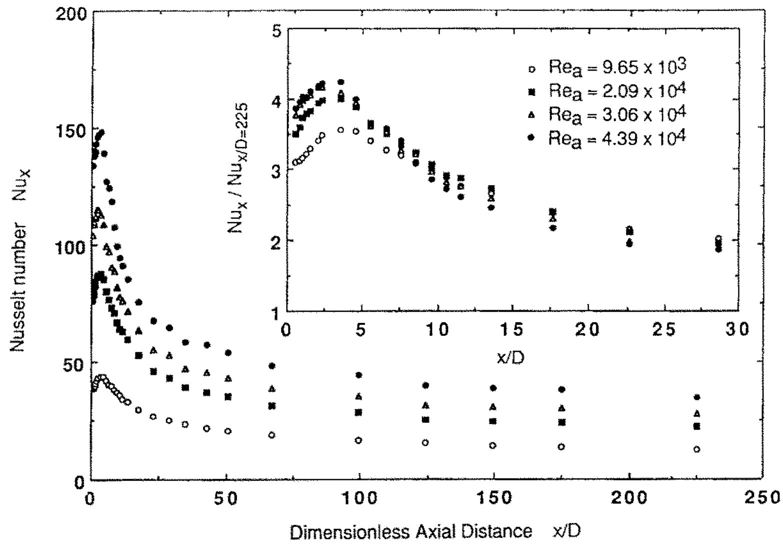

FIG. 10. Turbulent heat transfer results for 1000 ppm polyacrylamide solution ($D/d = 1.391$). Inset shows relative local Nusselt number vs x/D.

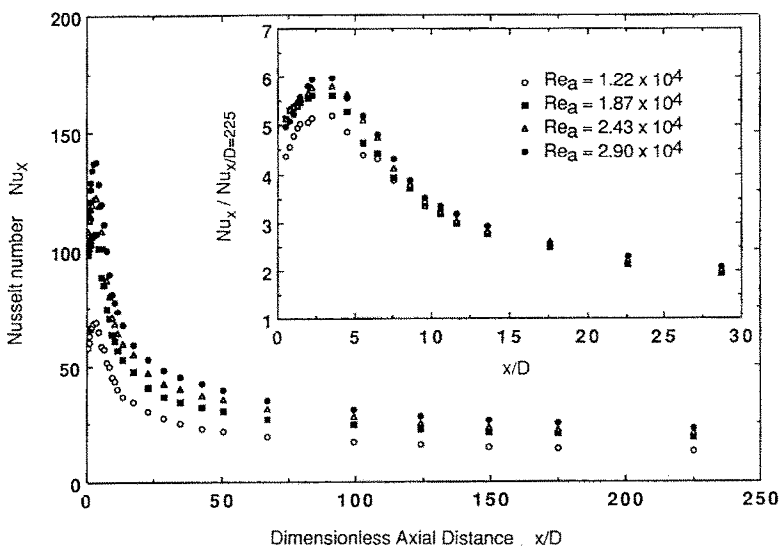

FIG. 11. Turbulent heat transfer results for 1000 ppm polyacrylamide solution ($D/d = 1.899$). Inset shows relative local Nusselt number vs x/D.

to drag-reducing viscoelastic fluid flows than to water flow.

The heat transfer test results for the 1000 ppm polyacrylamide solution at four different Reynolds numbers are presented in Figs. 10 and 11. Trends similar to those for the 200 ppm solution were revealed. The relative maximum Nusselt number seems to decrease as the concentration of the polyacrylamide solution increases. Additionally, the relative Nusselt number for the polyacrylamide solution approaches 2 at $x/D = 30$, which demonstrates that the local heat transfer coefficient should be two times greater than the fully developed value even at $x/D = 30$. Furthermore, the effect of the expansion ratio for the 1000 ppm solution can be seen from comparison of the insets in Figs. 10 and 11. The relative maximum Nusselt number is approximately 3.5–4.3 for $D/d = 1.391$, while it is approximately 5.2–6.0 for $D/d = 1.899$, indicating that the relative maximum Nusselt number increases with increasing expansion ratios for the 1000 ppm solution.

Figure 12 presents the local maximum Nusselt number as a function of the Reynolds number $Re_{a,d}$, which is based on the apparent viscosity η_a and the upstream diameter d. The solid line in Fig. 12 indicates the local maximum Nusselt number calculated from equation (3) for a Newtonian fluid [5]. The local maximum Nusselt number for the 200 ppm polyacrylamide solu-

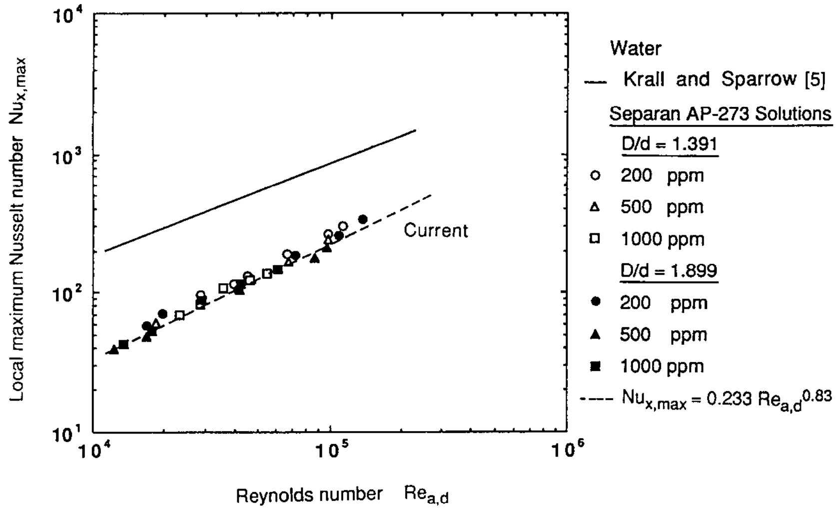

FIG. 12. Maximum Nusselt number vs Reynolds number for polyacrylamide solutions.

tion shows a significant drop from the Newtonian value at a given Reynolds number. Further increases in the polyacrylamide concentration do not result in subsequent decreases in the local maximum Nusselt number, suggesting that a minimum asymptote for the local maximum Nusselt number exists in the sudden-expansion pipe flow. A similar trend in the heat transfer reduction phenomenon of drag-reducing viscoelastic fluids is often observed in a straight pipe flow. This new asymptotic value of the local maximum Nusselt number in the sudden-expansion pipe flow for viscoelastic fluid flows is shown as a dashed line in Fig. 12 and can be shown by the following relationship:

$$Nu_{x,\max} = 0.233 Re_{a,d}^{0.83}. \quad (14)$$

This correlation was obtained by curve-fitting the local maximum Nusselt number data of the 500 and 1000 ppm polyacrylamide solutions.

To determine the extent of the heat transfer enhancement due to the sudden-expansion step in the separated, reattached, and redeveloping regions, the average Nusselt number, Nu_{ave}, is calculated by integrating the local Nusselt number over the entire heat transfer test section. Table 1 presents the average Nusselt number, the fully developed Nusselt number, and the percentage heat transfer enhancement due to the sudden-expansion step for water. The heat transfer enhancement due to the step for water is generally in the range of 3–6% from the fully developed value. It decreases slightly (from 5.3 to 4.7%) with increasing Reynolds number for $D/d = 1.391$ while the percentage enhancement decreases from 6% at $Re = 14\,700$ to 3.2% at $Re = 78\,700$ for $D/d = 1.899$, which indicates that the effect of the Reynolds number on the overall heat transfer performance is more sensitive in the larger expansion ratio case than in the smaller one.

Table 2 presents the percentage heat transfer enhancement for the polyacrylamide solutions. The major difference between the results with water and with viscoelastic fluids is the fact that the percentage heat transfer enhancement due to the sudden-expansion step for polyacrylamide solutions is in the range of 40–63% while that for water is in the range of 3–6%. Furthermore, the percentage heat transfer enhancement consistently increases with increasing expansion ratios, which is not the case for water. In addition, the effect of the Reynolds number on the percentage heat transfer enhancement is relatively minor for polyacrylamide solutions, while the percentage heat transfer enhancement has a major drop of 50% when Re increases from 14 700 to 78 700 for $D/d = 1.899$ for water.

Despite the fact that the percentage heat transfer enhancement, due to the sudden-expansion step, is much greater for the polyacrylamide solutions than for water, the absolute value of the average Nusselt number for water is still much larger than those for

Table 1. Average Nusselt number, fully developed Nusselt number, and percentage heat transfer enhancement in a circular pipe with a sudden-expansion step for water

D/d	Re	Nu_{ave}	Nu_{fd}	Enhancement (%)†
1.391	12 000	94.79	90.00	5.32
	32 000	231.34	216.10	4.55
	43 950	316.62	302.13	4.80
	64 300	439.72	419.09	4.92
	80 300	543.68	510.55	4.53
	96 400	626.04	589.66	4.72
1.899	14 700	110.87	104.58	6.05
	24 400	170.43	161.42	5.75
	36 300	252.61	242.40	4.01
	45 100	293.19	281.31	4.37
	62 200	393.30	379.22	4.10
	78 700	484.98	468.59	3.19

† Heat transfer enhancement is defined as $(Nu_{\text{ave}}/Nu_{\text{fd}}) \times 100$.

Table 2. Average Nusselt number and percentage heat transfer enhancement in a circular pipe with a sudden-expansion step for viscoelastic fluids (polyacrylamide Separan AP-273 solutions)

Test fluid	D/d	Re_a	Nu_{ave}	$Nu_{x/D=225}$	Enhancement (%)†
200 ppm solution	1.391	12 300	17.62	12.10	40.20
		29 900	31.76	21.64	46.77
		52 800	50.82	35.67	42.48
		79 400	60.76	42.31	43.61
		100 400	79.28	53.70	47.63
	1.899	15 000	21.35	14.47	47.53
		23 700	30.21	18.88	59.99
		35 100	41.82	26.77	56.23
		51 600	54.17	35.59	52.21
		59 700	61.21	39.14	56.37
500 ppm solution	1.391	12 000	20.06	12.87	55.81
		30 000	32.64	21.28	53.38
		61 900	53.39	34.40	55.20
		69 300	58.59	37.91	54.54
	1.899	9660	16.94	10.76	57.43
		24 600	28.61	18.16	57.54
		35 500	39.33	24.20	59.74
		51 800	55.26	33.81	63.46
1000 ppm solution	1.391	9650	19.03	12.41	53.28
		20 900	33.17	21.99	50.82
		30 400	41.17	27.66	48.85
		43 900	51.52	34.81	48.02
	1.899	12 200	20.94	13.24	58.12
		18 700	29.57	19.06	55.17
		24 300	33.75	21.11	59.85
		29 000	37.54	23.06	62.80

† Heat transfer enhancement is defined as $(Nu_{ave}/Nu_{fd}) \times 100$.

the polyacrylamide solutions in a circular pipe with a sudden-expansion step, as shown in Tables 1 and 2. Thus, it is clear that water cannot be replaced by a drag-reducing viscoelastic fluid in order to take advantage of the sudden-expansion step. However, many energy-intensive industries that do not use water in their heat transfer processes (such as the chemical, pharmaceutical, food, and biomedical industries) process a huge amount of viscoelastic fluids through various types of heat exchangers. It is with these industries that one can capitalize upon the enhancement of heat transfer performance due to the sudden-expansion step.

SUMMARY AND CONCLUSIONS

Aqueous solutions of polyacrylamide (Separan AP-273: 200, 500, and 1000 ppm) were used in the investigation of the heat transfer characteristics of drag-reducing viscoelastic fluids in a sudden-expansion pipe. The local Nusselt number in the sudden-expansion pipe flow was obtained as a function of the Reynolds number and the dimensionless axial distance from the sudden-expansion step. Important findings are briefly summarized below.

(1) A minimum asymptote of the local maximum Nusselt number exists at the sudden-expansion pipe flow with the drag-reducing viscoelastic fluids. A new correlation for this asymptote is proposed by the following equation:

$$Nu_{x,max} = 0.233\, Re_{a,d}^{0.83}.$$

(2) The values of the relative local maximum Nusselt number for the drag-reducing viscoelastic fluids are two to three times larger than those for the Newtonian fluid (water), which indicates that a greater heat transfer enhancement can be achieved due to the sudden-expansion step for the viscoelastic fluid flows than can be achieved with water.

(3) When the average Nusselt number is compared with and without the sudden-expansion step, the percentage heat transfer enhancement due to the sudden-expansion step for the drag-reducing viscoelastic fluids is in the range of 40–63% depending on Reynolds number, while that for water is in the range of 3–6%. Furthermore, the percentage heat transfer enhancement consistently increases with increasing expansion ratios for the drag-reducing viscoelastic fluids, while this is not the case for water.

(4) Heat transfer enhancement for the drag-reducing viscoelastic fluid extends to at least 250 pipe diameters, while that for water extends to only 15–20 pipe

diameter distances, validating the hypothesis that the long redeveloping region can be used for the heat transfer enhancement of drag-reducing viscoelastic fluids.

(5) Local maximum heat transfer due to the sudden-expansion step occurs at 1.0–1.80 pipe diameters from the step for the Newtonian fluid, while it occurs at 2.27–3.53 pipe diameters for the drag-reducing viscoelastic fluids. This suggests that the recirculating flow regime for the viscoelastic fluid flow is larger than that for the Newtonian fluid flow.

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ETUDE DU TRANSFERT THERMIQUE TURBULENT DANS L'ELARGISSEMENT BRUSQUE D'UN TUBE ET UN FLUIDE VISCOELASTIQUE REDUCTEUR DE PERTE DE CHARGE

Résumé—Le transfert thermique turbulent pour un fluide viscoélastique réducteur de perte de charge est étudié expérimentalement dans un tube à élargissement brusque avec des solutions aqueuses de polyacrylamide (200, 500 et 1000 ppm). Les domaines des nombres de Reynolds et de Prandtl sont respectivement de 6200–63 000 et 8,5–16,2. Une asymptote minimale du nombre de Nusselt local existe à l'élargissement brusque avec ce fluide et elle est donnée par l'équation suivante: $Nu_{x,\max} = 0{,}233 Re_{a,d}^{0,83}$. Quand le nombre de Nusselt moyen est comparé au cas sans élargissement brusque, le pourcentage d'accroissement du transfert dû à l'élargissement pour les solutions de polyacrylamide est de 40–63% alors qu'il n'est que de 3–6% pour l'eau. Cela suggère qu'il est possible pour beaucoup d'industries consommatrices d'énergie de pouvoir exploiter les performances accrues par les élargissements brusques.

UNTERSUCHUNG DES WÄRMEÜBERGANGS BEI TURBULENTER STRÖMUNG EINES WIDERSTANDSVERMINDERNDEN VISKOELASTISCHEN FLUIDS IN EINEM ROHR MIT PLÖTZLICHER EXPANSION

Zusammenfassung—Das Verhalten des turbulenten Übergangs in einem widerstandsvermindernden viskoelastischen Fluid wird experimentell anhand wässriger Lösungen von Polyacrylamid (200, 500 und 1000 ppm) untersucht. Die Reynolds-Zahl liegt dabei zwischen 6200 und 63 000, die Prandtl-Zahl zwischen 8,5 und 16,2. Es zeigt sich daß es eine untere Grenze für die maximale örtliche Nusselt-Zahl gibt, die mit folgender Gleichung beschrieben werden kann: $Nu_{x,max} = 0{,}233 Re_{a,d}^{0,83}$. Vergleicht man die mittlere Nusselt-Zahl mit dem Fall ohne plötzliche Expansion, so zeigt sich für Polyacrylamidlösungen eine Verbesserung um 40 bis 63% (abhängig von der Reynolds-Zahl), für Wasser nur 3 bis 6%. Die laufenden Ergebnisse deuten darauf hin, daß viele energieintensive Industrieunternehmen, welche große Mengen viskoelastischer Fluide mit Hilfe unterschiedlicher Typen von Wärmeaustauschern verarbeiten, aus der Verbesserung des Wärmeübergangs bei plötzlicher Expansion Nutzen ziehen können.

ИССЛЕДОВАНИЕ ТУРБУЛЕНТНОГО ТЕПЛОПЕРЕНОСА УПРУГОВЯЗКОЙ СНИЖАЮЩЕЙ СОПРОТИВЛЕНИЕ ЖИДКОСТИ ВО ВНЕЗАПНО РАСШИРЯЮЩЕЙСЯ ТРУБЕ

Аннотация—Турбулентный теплоперенос вязкоупругой снижающей сопротивление жидкости во внезапно расширяющейся круглой трубе экспериментально исследуется для водных растворов полиакриламида (200, 500 и 1000 ppm). Интервалы изменения чисел Рейнольдса и Прандтля составляют соответственно 6200–63 000 и 8,5–16,2. При течении в рассматриваемых условиях обнаружено существование минимальной асимптоты локального максимального числа Нуссельта, которая описывается уравнением $Nu_{x,\,max} = 0{,}233 Re_{a,\,d}^{0,83}$. При сравнении среднего числа Нуссельта в случаях с внезапным расширением и без него выявлено, что для полиакриламидных растворов доля увеличения теплопереноса из-за резкого расширения составляет 40–63% в зависимости от числа Рейнольдса, в то время как для воды—3–6%. На основе полученных результатов можно предположить, что, во многих энергоемких отраслях промышленности, обрабатывающих большие количества вязкоупругих жидкостей с применением теплообменников различных типов, выгодна интенсификация теплопереноса с использованием ступени внезапного расширения.